Assignment on Hoare Logic

1. Please give a formal proof, using the Hoare logic rules, of the following partial-correctness specification.

   $$\begin{array}{l} \{x = 0\} \\ \textbf{while } x < 100 \textbf{ do} \\ \quad x := x + 1; \\ \quad y := x; \\ \{x = 100 \wedge y = 100\} \end{array}$$

   Also, please write down the loop invariant in your proof.

2. Please give a formal proof, using the Hoare logic rules, of the following total-correctness specification.

   $$\begin{array}{l} [y > 0] \\ r := x; \\ z := 0; \\ \textbf{while } y \leq r \textbf{ do} \\ \quad r := r - y; \\ \quad z := z + 1; \\ [r < y \wedge x = r + y * z] \end{array}$$

   In the class, we say that the loop invariant is $(x = r + y * z) \wedge y > 0$, and the loop variant is r. Does your proof use them?

3. (a) Consider Hoare triples of the form $\{\textbf{true}\}x := e\{x = e\}$.
      - i. Write down an instance of such a triple that cannot be proved using Hoare logic and explain why not.
      - ii. Write down conditions on x and e such that $\{\textbf{true}\}x := e\{x = e\}$ can be proved and give a proof of this assuming your conditions.

   (b) Consider Hoare triples of the form $[\textbf{true}]c[\textbf{true}]$. Write down an instance of such a triple that cannot be proved using Hoare logic and explain why not.

4. In this problem we add the "repeat" command to the simple imperative language. We extend the syntax as follows:

$$(\mathit{Comm}) \quad c \quad ::= \quad \ldots \mid \textbf{repeat}\ c\ \textbf{until}\ b$$

The meaning of **repeat** c **until** b is that c is executed and then b is tested; if the result is **true**, then nothing more is done, otherwise the whole **repeat** command is repeated. Thus **repeat** c **until** b is equivalent to c **;** **while** $\neg b$ **do** c.

Give the partial correctness Hoare logic rule for **repeat** c **until** b.