# Assignment on Mathematical Background

For each of the following statements, is it true? If your answer is yes, just say yes. If your answer is no, give a counterexample.

For instance, this simple statement is not true: $\forall x, y \in \mathbf{B}.\ x = y$. To give a counterexample, we instantiate $x$ and $y$ so that $x, y \in \mathbf{B}$ but $x \neq y$. So a counterexample can be: let $x$ be **true** and $y$ be **false**.

1. Recall $\mathcal{P}(S)$ is the powerset of $S$. Then,

$$\forall f.\ f \in \mathbf{N} \to \mathbf{N} \implies \exists A, B.\ (A \subseteq \mathbf{N}) \wedge (B \subseteq \mathbf{N}) \wedge (\mathcal{P}(f) \subseteq A \to B).$$

2. We define the relation $\bowtie$ between two functions $f, g \in \mathbf{N} \to \mathbf{N}$ as follows:

$$f \bowtie g \ \text{ iff } \ \forall x, y.\ (f(x) = 42) \wedge (g(y) = 42) \implies (x = y).$$

   Then $\bowtie$ is transitive, that is,

$$\forall f, g, h.\ (f \bowtie g) \wedge (g \bowtie h) \implies (f \bowtie h).$$

3. Let

$$H \ = \bigcup_{S \subseteq^{\text{fin}} \mathbf{N}} (S \to \mathbf{N}).$$

   Then

$$\forall h_1, h_2.\ (h_1 \in H) \wedge (h_2 \in H) \implies (h_1 \cup h_2 \in H).$$

4. Again, let $H = \bigcup_{S \subseteq^{\text{fin}} \mathbf{N}} (S \to \mathbf{N})$. We also define $\mathsf{closed}(s, h)$ and $\mathsf{hclosed}(h)$ for $s \in \mathcal{P}(\mathbf{N})$ and $h \in H$ as follows:

$$\begin{array}{lll} \mathsf{closed}(s, h) & \text{iff} & \forall l, l'.\ (l \in s) \wedge (l' = h(l)) \implies l' \in s \\ \mathsf{hclosed}(h) & \text{iff} & \mathsf{closed}(\mathrm{dom}(h), h) \end{array}$$

   Then

$$\forall h \in H.\ (\forall x.\ \mathsf{hclosed}(h\{x \rightsquigarrow 42\})) \implies \mathsf{hclosed}(h).$$